

GROUP A. Postulates for the predicate calculus.

GROUP A1. Postulates for the propositional calculus.

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| 1a. $A \supset (B \supset A)$. | 2. $\frac{A, A \supset B}{B}$. |
| 1b. $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$. | 4a. $A \& B \supset A$. |
| 3. $A \supset (B \supset A \& B)$. | 4b. $A \& B \supset B$. |
| 5a. $A \supset A \vee B$. | 6. $(A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C))$. |
| 5b. $B \supset A \vee B$. | 8°. $\neg \neg A \supset A$. |
| 7. $(A \supset B) \supset ((A \supset \neg B) \supset \neg A)$. | |

GROUP A2. (Additional) Postulates for the predicate calculus.

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| 9. $\frac{C \supset A(x)}{C \supset \forall x A(x)}$. | 10. $\forall x A(x) \supset A(t)$. |
| 11. $A(t) \supset \exists x A(x)$. | 12. $\frac{A(x) \supset C}{\exists x A(x) \supset C}$. |

GROUP B. (Additional) Postulates for number theory.

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| 13. $A(0) \& \forall x(A(x) \supset A(x')) \supset A(x)$. | 15. $\neg a' = 0$. |
| 14. $a' = b' \supset a = b$. | 17. $a = b \supset a' = b'$. |
| 16. $a = b \supset (a = c \supset b = c)$. | 19. $a + b' = (a + b)'$. |
| 18. $a + 0 = a$. | 21. $a \cdot b' = a \cdot b + a$. |
| 20. $a \cdot 0 = 0$. | |

(The reason for writing “°” on Postulate 8 will be given in § 23.)

One may verify that 14—21 are formulas; and that 1—13 (or in the case of 2, 9 and 12, the expression(s) above, and the expression below, the line) are formulas, for each choice of the A , B , C , or x , $A(x)$, C , t , subject to the stipulations given at the head of the postulate list.

The class of ‘axioms’ is defined thus. A formula is an *axiom*, if it has one of the forms 1a, 1b, 3—8, 10, 11, 13 or if it is one of the formulas 14—21.

The relation of ‘immediate consequence’ is defined thus. A formula is an *immediate consequence* of one or two other formulas, if it has the form shown below the line, while the other(s) have the form(s) shown above the line, in 2, 9 or 12.

This is the basic metamathematical definition corresponding to Postulates 2, 9 and 12, but we shall restate it with additional terminology